

# TECHNICAL REPORT



**Multicore and symmetrical pair/quad cables for digital communications –  
Part 1-3: Electrical transmission parameters for modelling cable assemblies  
using symmetrical pair/quad cables**

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## INTERNATIONAL ELECTROTECHNICAL COMMISSION

**MULTICORE AND SYMMETRICAL PAIR/QUAD  
CABLES FOR DIGITAL COMMUNICATIONS –****Part 1-3: Electrical transmission parameters for modelling cable  
assemblies using symmetrical pair/quad cables**

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The text of this technical report is based on the following documents:

Enquiry draft	Report on voting
46C/924/DTR	46C/932/RVC

Full information on the voting for the approval of this technical report can be found in the report on voting indicated in the above table.

This publication has been drafted in accordance with the ISO/IEC Directives, Part 2.

A list of all the parts in the IEC 61156 series, published under the general title *Multicore and symmetrical pair/quad cables for digital communications*, can be found on the IEC website.

The committee has decided that the contents of this publication will remain unchanged until the stability date indicated on the IEC web site under "<http://webstore.iec.ch>" in the data related to the specific publication. At this date, the publication will be

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## MULTICORE AND SYMMETRICAL PAIR/QUAD CABLES FOR DIGITAL COMMUNICATIONS –

### Part 1-3: Electrical transmission parameters for modelling cable assemblies using symmetrical pair/quad cables

#### 1 Scope

This technical report is a supplement to IEC 61156-1 Edition 3 (2007): Multicore and symmetrical pair/quad cables for digital communications – Part 1: Generic specification.

This technical report covers the following topics following this standard:

- the near-end crosstalk test methods and length correction procedures of 6.3.5;
- the far-end crosstalk test methods and length correction procedures of 6.3.6;
- the concatenation of measured cable segments, even if they are of different design.

The final objective of this technical report is to describe the mathematics involved to model the concatenation of cable sections of different length, not based upon measurements but based upon the specification limits of the cables involved. This is required as a base foundation of the complete channel modelling, involving also the connectivity covered by IEC SC48B towards channels, as required and requested by ISO/IEC JTC1/SC25 WG3 for incorporation into ISO/IEC 11801:2002 [1]<sup>1</sup>.

This TR is informative and contains observations and recommendations applicable to using the length correction formulas for either measurements or modelling of balanced cables.

#### 2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IEC 60050-726, *International Electrotechnical Vocabulary – Part 726: Transmission lines and waveguides*

IEC 61156-1:2007, *Multicore and symmetrical pair/quad cables for digital communications – Part 1: Generic specification*

IEC/TR 61156-1-2, *Multicore and symmetrical pair/quad cables for digital communications – Part 1-2: Electrical transmission characteristics and test methods of symmetrical pair/quad cables*

IEC 61156-5, *Multicore and symmetrical pair/quad cables for digital communications – Part 5: Symmetrical pair/quad cables with transmission characteristics up to 1 000 MHz – Horizontal floor wiring – Sectional specification*

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<sup>1</sup> The figures in square brackets refer to the Bibliography.

IEC 61156-6, *Multicore and symmetrical pair/quad cables for digital communications – Part 6: Symmetrical pair/quad cables with transmission characteristics up to 1 000 MHz – Work area wiring – Sectional specification*

IEC/TR 62152, *Transmission properties of cascaded two-ports or quadripols – Background of terms and definitions*

### 3 Terms, definitions, symbols, units and abbreviated terms

#### 3.1 Terms and definitions

For the purposes of this document, the terms and definitions given in IEC 60050-726, IEC/TR 61156-1-2, and IEC/TR 62152 apply.

#### 3.2 Symbols, units and abbreviated terms

For the purposes of this document, the following symbols, units and abbreviated terms apply.

Transmission line equation electrical symbols and related terms and symbols:

$R$	pair resistance ( $\Omega/\text{m}$ )
$L$	pair inductance ( $\text{H}/\text{m}$ )
$G$	pair conductance ( $\text{S}/\text{m}$ )
$C$	pair capacitance ( $\text{F}/\text{m}$ )
$\alpha$	attenuation coefficient ( $\text{Np}/\text{m}$ , or $\text{dB}$ as indicated)
$\beta$	phase coefficient ( $\text{rad}/\text{m}$ )
$\gamma$	propagation coefficient ( $\text{Np}/\text{m}$ , $\text{rad}/\text{m}$ )
$x$	length coordinate ( $\text{m}$ )
$Z_0$	complex characteristic impedance, or mean characteristic impedance if the pair is homogeneous or free of structure (also used to represent a function fitted result) ( $\Omega$ )
$l$	length, variable ( $\text{m}$ )
$M$	length, reference, disturbing ( $\text{m}$ )
$A$	length, reference, disturbed ( $\text{m}$ )
$j$	imaginary denominator
$\omega$	radian frequency ( $\text{rad}/\text{s}$ )
$f$	frequency ( $\text{Hz}$ )
$I$	current, coupled
$I_{\text{diff}}$	current in the differential-mode circuit ( $\text{I}$ )
$I_{\text{com}}$	current in the common-mode circuit ( $\text{I}$ )
$U_{\text{diff}}$	voltage in the differential-mode circuit ( $\text{V}$ )
$U_{\text{com}}$	voltage in the common-mode circuit ( $\text{V}$ )
1, 2	index to designate the pair 1 and pair 2, respectively
$N, F$	index to designate the near end and far end, respectively
$TU$	transverse unbalance
$LU$	longitudinal unbalance
$K$	coupling coefficient
$K_N$	near end cross-talk coupling coefficient
$K_F$	far end cross-talk coupling coefficient



$k_1, k_2, k_3$	attenuation coefficients for the twisted pair
$FEXT$	far-end crosstalk loss (dB)
$NEXT$	near-end crosstalk loss (dB)
$EL FEXT$	equal-level far-end crosstalk loss (dB)
$ACR-F$	attenuation-to-crosstalk-ratio far-end loss (dB)
$\Delta$	length correction coefficient
$S$	S parameter matrix
$S_{11}$	S parameter
$T$	T parameter matrix
$T_{11}$	T parameter
ab	index to designate the incident port and reflected port, of multiport parameter

## 4 Traditional length correction formulae

### 4.1 Introduction

The traditional length correction formulae were intended for measurements on long manufactured lengths to be corrected to the specified nominal length; i.e. for cables complying to IEC 61156-5 and IEC 61156-6, as outlined in IEC 61156-1. Therein the length corrections apply to measurements made on longer lengths than 100 m, to be corrected to the 100 m specification. Moreover, these formulae were normally used in the cable industry for quality assurance purposes.

The formulae are intended for measurements of crosstalk within cables with length uncorrelated crosstalk coupling characteristics. Thus they do not readily adapt to the limit lines for crosstalk loss, which are the upper-bounds for the characteristic length correlated crosstalk coupling, i.e. a homogeneous coupling along a cable that is at the limit line at every frequency, at the specified length.

### 4.2 Length correction formulae in IEC 61156-1

The formulae are

$$FEXT_{\ell} = FEXT_M - 10 \cdot \log_{10} \left( \frac{\ell}{M} \right) - \alpha_M + \alpha_{\ell} \quad (1)$$

and

$$NEXT_{\ell} = NEXT_M - 10 \cdot \log_{10} \left( \frac{1 - 10^{-\frac{4\alpha_{\ell}}{20}}}{1 - 10^{-\frac{4\alpha_M}{20}}} \right) \quad (2)$$

where

- $\ell$  is the actual cable conversion length;
- $M$  is the reference cable specification length;
- $\alpha$  is the attenuation for the indexed length in dB.

Normally, we measure FEXT and derive from it, using the corresponding attenuation, either the EL FEXT or more pertinent to data grade cables the ACR-F.

For these last two values, we have then the following length corrections:

$$EL\ FEXT_{\ell} = EL\ FEXT_M - 10 \cdot \log_{10} \left( \frac{\ell}{M} \right) \quad (3)$$

and

$$ACR - F_{\ell} = ACR - F_{\Lambda} - 10 \cdot \log_{10} \left( \frac{\ell}{\Lambda} \right) \quad (4)$$

Here a distinction between the length  $M$  and  $\Lambda$  is made to indicate the difference between disturbing and disturbed pair attenuation, respectively.

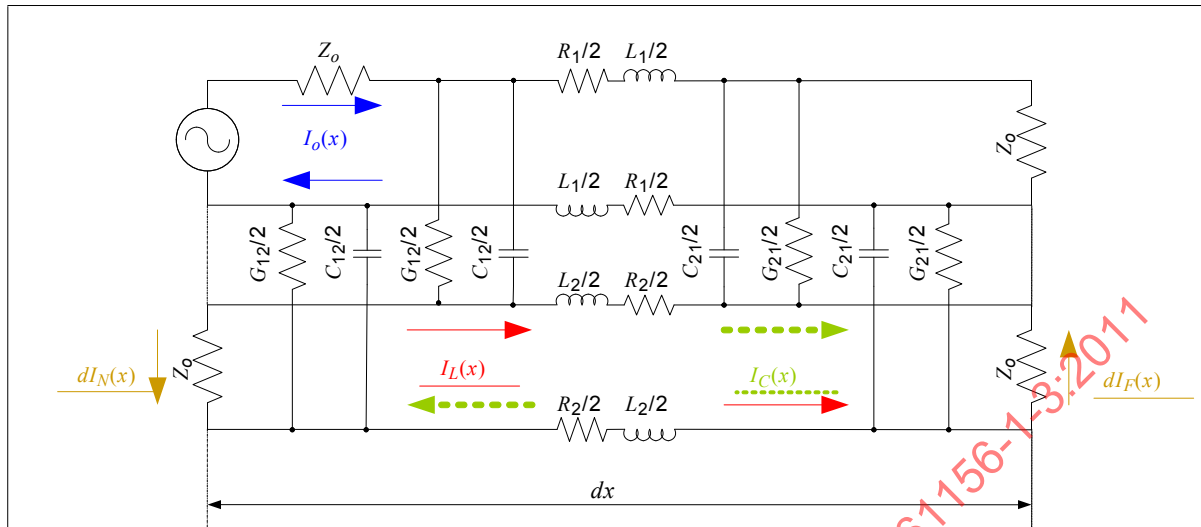
The measurement magnitude values or the complex values of the actual cable may be used to compute the crosstalk parameter when applying the traditional length correction formula, though these formulae refer only to magnitude values.

#### 4.3 The development of the traditional cross-talk length correction formulae NEXT and EL FEXT [3]

First only in-put to out-put and the out-put to out-put cross-talk coupling are considered. These correspond to the near-end cross-talk and the equal level far-end cross-talk. These are called in the cable industry generally NEXT (IO-NEXT though this denomination is in the present case irrelevant) and EL FEXT (or OO-FEXT). These two terms are treated first, before going over to the in-put to out-put FEXT (IO-FEXT).

NOTE It should be noted that the following derivation was first published by the members of the technical staff of the Bell telephone laboratories [6].

If we consider the coupling between two infinitesimal short circuits, we have to take first the unbalances of the primary parameters of both circuits 1 and 2 into account. This inherently implies the assumption that the primary parameters as prime responsible factor for the crosstalk coupling are statistically distributed over the length of the cable.



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**Key**

$I_C(x)$	current induced at the length $x$ due to capacitive coupling
$I_o(x)$	current going into the infinitesimal length of the line $dx$ at the length $x$
$I_L(x)$	current induced at the length $x$ due to inductive coupling
$dI_N(x)$	current increment flowing through the near end termination of the infinitesimal length element
$dI_F(x)$	current increment flowing through the far end termination of the infinitesimal length element
$Z_o$	impedance of the termination of the length element. It is assumed here to be identical for all source and load impedances, and corresponds additionally to the characteristic impedance of the pairs

**Figure 1 – Coupling between two circuits due to unbalances of the primary parameters**

We get then according to Figure 1 for the corresponding crosstalk values of interest between two infinitesimally short circuits:

As a result of the above, it is implied that the integration direction of the infinitesimal current or voltage increments is reversed in direction.

Besides the mathematically easier treatment, this has also an historical background. Thus the telephone linesmen could not determine the IO-FEXT, but they could easily measure the OO-FEXT on the poles.

For the transverse and the longitudinal unbalances of the primary parameters, we get following the indications in Figure 1:

$$TU = (G_{21} + j \cdot \omega \cdot C_{21}) - (G_{12} + j \cdot \omega \cdot C_{12}) \quad (5)$$

$$LU = (R_2 + j \cdot \omega \cdot L_2) - (R_1 + j \cdot \omega \cdot L_1) \quad (6)$$

where

$TU$  is the transverse unbalance between the pairs of the corresponding primary parameters  $G$  and  $C$ ;

$LU$  is the longitudinal unbalance between the pairs of the corresponding primary parameters  $R$  and  $L$ ;

- 1,2 are indices indicating pair 1 and 2;  
 $G$  is the conductance unbalance between the pairs;  
 $C$  is the capacitance unbalance between the pairs;  
 $R$  is the mutual resistance unbalance of the pairs;  
 $L$  is the mutual inductance unbalance of the pairs;  
 $j$  is the complex operator;  
 $\omega$  is the circular frequency.

We neglect the conductance unbalance between the pairs which we can – at least for modern data grade cables – assume to be zero. This is the result of the use of insulating materials with a very low  $\tan\delta$ , like PE or FEP. In fact, the resulting conductance unbalance is generally so small that it would be extremely hard to determine it at all.

We then get

$$G_{12} = G_{21} \approx 0 \quad (7)$$

$$TU = j \cdot \omega \cdot C_{21} - j \cdot \omega \cdot C_{12} = j \cdot \omega \cdot (C_{21} - C_{12}) \quad (8)$$

$$LU = (R_2 - R_1) + j \cdot \omega \cdot (L_2 - L_1) \quad (9)$$

We can furthermore assume that both infinitesimal elements of Figure 1 are on each side terminated in  $Z_o$ , which is also the characteristic impedance of the pairs considered. In other words, we consider only the case of perfectly matched pairs. The impedance of the capacitance unbalances is as a result much higher than the characteristic impedance, such that we may neglect the latter one to calculate the current going through each termination. In this case – due to the fact of matched impedances – we have then for the infinitesimal element the transverse and the longitudinal unbalances of the primary parameters of the pairs considered:

We then get

$$2 \cdot I_C(x) = -j \cdot \omega \cdot \frac{C_{12} - C_{21}}{2} \cdot \frac{Z_o \cdot I_o(x)}{2} \quad (10)$$

and

$$I_L(x) = -\frac{R_2 - R_1}{2 \cdot Z_o} - j \cdot \omega \cdot \frac{L_1 - L_2}{2 \cdot Z_o} \quad (11)$$

or with

$$C = C_{12} - C_{21} \quad (12)$$

$$R = R_2 - R_1 \quad (13)$$

$$L = L_1 - L_2 \quad (14)$$

we get

$$I_C(x) = -j \cdot \omega \cdot \frac{C \cdot Z_o \cdot I_o(x)}{8} \quad (15)$$

and

$$I_L(x) = - \left( \frac{R}{2 \cdot Z_o} + j \cdot \omega \cdot \frac{L}{2 \cdot Z_o} \right) \cdot I_o(x) \quad (16)$$

In a further step, we can neglect also the longitudinal resistance unbalance between the pairs, i.e. we assume  $R \approx 0$ . This is definitely acceptable for modern data grade cables.

However, in the past, this approximation was only justifiable on a large scale statistical basis. The main reason for this was the fact that frequently, the tangential line between the twisted conductors was – due to tension unbalances in the twisters – not a straight line. In other words, one conductor is more or less wrapped around the other wire. This resulted of course in the result that one wire was longer than the other, and there resulted a relatively high resistance unbalance which definitely affected severely the cross-talk.

For the currents at the near end and the far end of the infinitesimal element, we then get

$$dI_N(x) = -j \cdot \omega \cdot \left( \frac{C \cdot Z_o}{8} + \frac{L}{2 \cdot Z_o} \right) \cdot I_N(x) \quad (17)$$

and

$$dI_F(x) = -j \cdot \omega \cdot \left( \frac{C \cdot Z_o}{8} - \frac{L}{2 \cdot Z_o} \right) \cdot I_F(x) \quad (18)$$

where

$I_N(x)$  is the current flowing through the near end termination of the length element at the termination point  $x$ , i.e. just before the infinitesimal length element as seen from the right side;

$I_F(x)$  is the current flowing through the far end termination point  $(\ell-x)$ , i.e. just before the infinitesimal length element as seen from the left side;

$x$  is the length coordinate of the considered cable element.

If we use the abbreviations

$$K_N = \left( \frac{Z_o \cdot C}{8} + \frac{L}{2 \cdot Z_o} \right) \quad (19)$$

$$K_F = \left( \frac{Z_o \cdot C}{8} - \frac{L}{2 \cdot Z_o} \right) \quad (20)$$

we get for the near end and far end currents, respectively, taking additionally the propagation constants of each pair into account:

$$I_N(x) = I_o \cdot K_N \cdot e^{-(\gamma_1 + \gamma_2) \cdot x} \quad (21)$$

$$I_F(x) = I_o \cdot K_F \cdot e^{-(\gamma_1 + \gamma_2) \cdot (\ell - x)} \quad (22)$$

where

$\gamma_1$  is the propagation constant of the first pair;  
 $\gamma_2$  is the propagation constant of the second pair.

We can now determine the current ratio representing the current ratio of the near- and far-end cross-talk coupling:

We have

$$\frac{dI_N(x)}{I_o} = -j \cdot \omega \cdot K_N \cdot e^{-(\gamma_1 + \gamma_2) \cdot x} \quad (23)$$

$$\frac{dI_F(x)}{I_o} = -j \cdot \omega \cdot K_F \cdot e^{-(\gamma_1 + \gamma_2) \cdot (\ell - x)} \quad (24)$$

If the cross-talk is uncorrelated with the length, then we can integrate Equations (23), (24) to calculate the crosstalk. We then obtain

$$\int dNEXT = \left( \frac{I_N(0)}{I_o} \right)^2 = \omega^2 \cdot K_N^2 \cdot \int_{x=0}^{x=\ell} e^{-(\gamma_1 + \gamma_2) \cdot x} \cdot e^{-(\gamma_1^* + \gamma_2^*) \cdot x} \cdot dx \quad (25)$$

$$\int dNEXT = \omega^2 \cdot K_N^2 \cdot \frac{1 - e^{-2 \cdot (\alpha_1 + \alpha_2) \cdot \ell}}{(\alpha_1 + \alpha_2) \cdot \ell} = 4 \cdot \pi^2 \cdot K_N^2 \cdot f^2 \cdot \frac{1 - e^{-2 \cdot (\alpha_1 + \alpha_2) \cdot \ell}}{(\alpha_1 + \alpha_2) \cdot \ell} \quad (26)$$

$$\int dELFEXT = \left( \frac{I_F(\ell)}{I_o} \right)^2 = \omega^2 \cdot K_F^2 \cdot \int_{x=0}^{x=\ell} e^{-(\gamma_1 - \gamma_2) \cdot (\ell - x)} \cdot e^{-(\gamma_1^* - \gamma_2^*) \cdot (\ell - x)} \cdot dx \quad (27)$$

$$\int dELFEXT = \omega^2 \cdot K_F^2 \cdot \ell = 4 \cdot \pi^2 \cdot K_F^2 \cdot \ell \cdot f^2 \quad (28)$$

where

$dNEXT$  is the increment of the NEXT ration due to the element  $dx$ ;

$dELFEXT$  is the increment of the FEXT ration due to the element  $dx$ ;

$\alpha_1$  is the attenuation of pair 1;

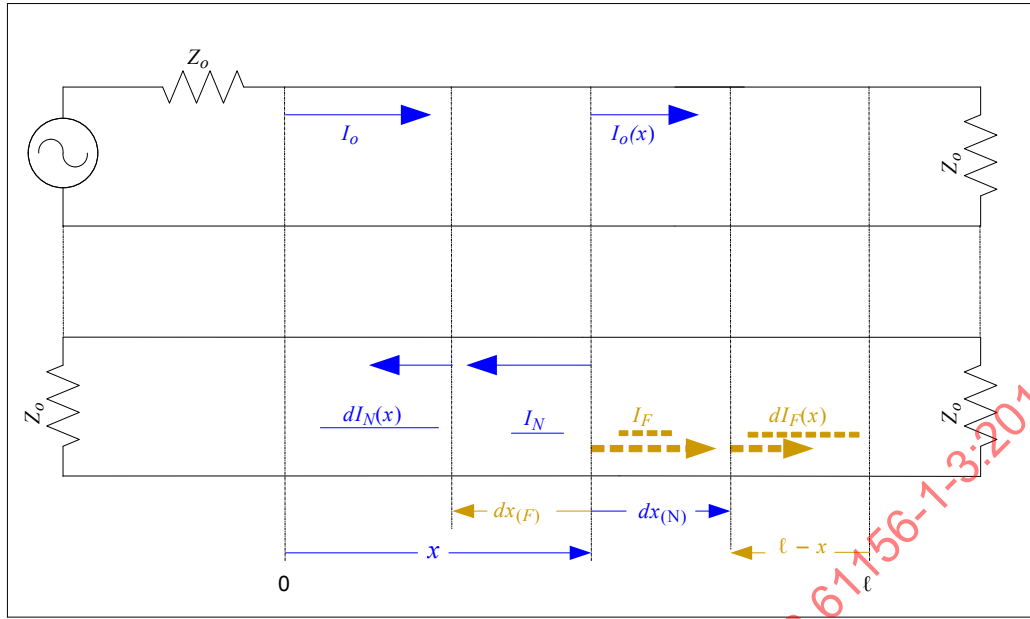
$\alpha_2$  is the attenuation of pair 2;

$f$  is the frequency;

$\ell$  is the length of the cross-talk coupled pairs;

$\otimes$  indicates that the conjugate complex has to be taken to calculate the square of a complex number.

Note that we were reversing the direction of the integration in the last case, see also Figure 2 and get as a result the equal level far-end cross-talk.



IEC 632/11

**Figure 2 – Integration of the coupled near- and far-end currents over the length of the cable**

Equations (26) and (28) are simplified more if we assume equal attenuations for the disturbing and the disturbed pair <sup>2</sup>. We have then

$$\alpha = \alpha_1 = \alpha_2 \quad (29)$$

$$NEXT(\ell, f) = \frac{\pi^2 \cdot f^2 \cdot K_N^2}{\alpha \cdot \ell} \cdot (1 - e^{-4 \cdot \alpha \cdot \ell}) \quad (30)$$

$$EL FEXT(\ell, f) = 4 \cdot \pi^2 \cdot K_F^2 \cdot \ell \cdot f^2 \quad (31)$$

If we want to express NEXT and EL FEXT as functions of the frequency and length, we get with the following constants:

$$C_N = \pi^2 \cdot K_N^2 \quad (32)$$

$$C_F = 4 \cdot \pi^2 \cdot K_F^2 \quad (33)$$

$$NEXT(\ell, f) = \frac{C_N \cdot f^2}{\alpha \cdot \ell} \cdot (1 - e^{-4 \cdot \alpha \cdot \ell}) \quad (34)$$

$$EL FEXT(\ell, f) = C_F \cdot \ell \cdot f^2 \quad (35)$$

Obviously in Equations (34), (35), only the attenuation is calculated in Neper per length, whereas the entire formulae are in absolute values. To convert them into decibels (dB), we use the following formula:

<sup>2</sup> It should be mentioned however, that the modelling using different propagation constants is also feasible.

$$\begin{aligned} NEXT(\ell, f) = & -10 \cdot \log_{10}(C_N) - 20 \cdot \log_{10}(f) + 20 \cdot \log_{10}(\ell) \\ & + 10 \cdot \log_{10}(\alpha) - 10 \cdot \log_{10}(1 - e^{-4 \cdot \alpha \cdot \ell}) \end{aligned} \quad (36)$$

$$\begin{aligned} ELFEXT(\ell, f) = & -10 \cdot \log_{10}(C_F) - 20 \cdot \log_{10}(f) \\ & - 10 \cdot \log_{10}(\ell) \end{aligned} \quad (37)$$

If we know now the function relating the attenuation to the frequency, we can simplify Equations (34), (35):

We have

$$\alpha \cdot \ell = k_1 \cdot \sqrt{f} \quad [\text{Neper at Length } \ell] \quad (38)$$

where  $k_1$  is a heuristic parameter to approximate in a simplified way the attenuation <sup>3</sup> as compared to Equation (40).

We have then for NEXT as a function of cable length and frequency

$$NEXT(\ell, f) = \frac{C_N \cdot f^{\frac{3}{2}}}{k_1} \cdot (1 - e^{-4 \cdot \alpha \cdot \ell}) \quad (39)$$

The attenuation in the last expression is in Neper.

If we use the usual formula for the attenuation, taking the low- and high-end attenuation effects due to the primary parameters into account

$$\alpha = k_1 \cdot \sqrt{f} + k_2 \cdot f + \frac{k_3}{\sqrt{f}} \quad (40)$$

where  $k_1$ ;  $k_2$ ;  $k_3$  are heuristic parameters to approximate the attenuation.

We get

$$NEXT(\ell, f) = \frac{C_N \cdot f^2 \cdot \sqrt{f}}{k_1 \cdot f + k_2 \cdot f \cdot \sqrt{f} + k_3} \cdot (1 - e^{-4 \cdot \alpha \cdot \ell}) \quad (41)$$

In general, Equation (38) is considered precise enough for length corrections of NEXT, so that we do not have to use Equation (40).

If we convert Equation (39) into decibels, then we get the well known relationship for NEXT <sup>4</sup>

<sup>3</sup> The error made using Equation (38) is minimal, but renders the length correction formula extremely complex, without contributing in any substantial way to the result. Hence, the last two expressions in these heuristic formulae can be neglected without making any noticeable error.

<sup>4</sup> Note that in the last term, the attenuation has still to be given in Neper per length.



$$NEXT(L, f) = -10 \cdot \log_{10} \left( \frac{C_N}{k_1} \right) - 15 \cdot \log_{10}(f) - 10 \cdot \log_{10} \left( 1 - e^{-4 \cdot \alpha \cdot \ell} \right) \quad [\text{dB at Length } \ell] \quad (42)$$

In Equation (42), the first term contains a constant. Normally, the last term can be neglected if the attenuation exceeds 20 dB. However, for channel simulations, this term cannot be neglected, as relatively short cables are frequently used. Equation (42) can be further rationalized, using the transformation modulus for logarithms. We have then to express all values in this equation in decibels:

$$NEXT(L, f) = -10 \cdot \log_{10} \left( \frac{C_N}{k_1} \right) - 15 \cdot \log_{10}(f) - 10 \cdot \log_{10} \left( 1 - 10^{-\frac{\alpha \cdot \ell}{5}} \right) \quad [\text{dB at Length } \ell] \quad (43)$$

We could also use Equation (40) in a similar way, though the result is substantially more complex at essentially only a negligible gain in precision. Therefore, this derivation is not indicated, as its use is not required.

For the ratio of NEXT of two different lengths of cables, we get then

$$NEXT(\ell, f) = NEXT(\Lambda, f) + 10 \cdot \log_{10} \left( \frac{1 - e^{-4 \cdot \alpha \cdot \Lambda}}{1 - e^{-4 \cdot \alpha \cdot \ell}} \right) \quad [\text{dB at Length } \ell] \quad (44)$$

$$NEXT(\ell, f) = NEXT(\Lambda, f) + 10 \cdot \log_{10} \left( \frac{1 - 10^{-\frac{\alpha \cdot \Lambda}{5}}}{1 - 10^{-\frac{\alpha \cdot \ell}{5}}} \right) \quad [\text{dB at Length } \ell] \quad (45)$$

where

$\Lambda$  is the cable length to which the cross-talk has to be corrected.

These are the generally accepted length correction formulae for NEXT. In the first equation of Equation (44), the attenuation has to be given in Neper per m, whereas in Equation (45), all values are to be given in decibels.

For the EL FEXT, the relationship is substantially easier. We use Equation (35) and get then for two different cable lengths <sup>5</sup>

$$ELFEXT(\ell, f) = ELFEXT(\Lambda, f) - 10 \cdot \log_{10}(C_F) + 10 \cdot \log_{10}(C_F) - 20 \cdot \log_{10}(f) + 20 \cdot \log_{10}(f) - 10 \cdot \log_{10}(\ell) + 10 \cdot \log_{10}(\Lambda) \quad (46)$$

$$ELFEXT(\ell, f) = ELFEXT(\Lambda, f) + 10 \cdot \log_{10} \left( \frac{\Lambda}{\ell} \right) \quad (47)$$

<sup>5</sup>

It should be kept in mind however, that deriving this equation, we have assumed that the cross-talk contributions from each infinitesimal section are completely uncorrelated to each other.

The EL FEXT is defined as the out-put to out-put far end cross-talk (OO-FEXT) (this fact rendered it mandatory for data grade cables to define ACR-F). As a result, we can calculate the FEXT as the difference between the EL FEXT and the attenuation <sup>6</sup>.

Hence we get for the in-put to out-put FEXT (IO-FEXT):

$$FEXT(\ell, f) = EL FEXT(\ell, f) - \alpha(\ell, f) \quad [\text{dB at } 100 \text{ m}] \quad (48)$$

With this formula in conjunction with Equation (47), we can derive the length correction for FEXT.

We get

$$FEXT(\ell, f) = FEXT(\Lambda, f) - \alpha(\ell, f) - \alpha(\Lambda, f) + 10 \cdot \log_{10} \left( \frac{\Lambda}{\ell} \right) \quad [\text{dB at length}] \quad (49)$$

or with

$$\alpha(\ell, f) = \alpha(\Lambda, f) \cdot \frac{\ell}{\Lambda} \quad (50)$$

$$FEXT(\ell, f) = FEXT(\Lambda, f) - \alpha(\Lambda, f) \cdot \left( \frac{\ell}{\Lambda} - 1 \right) + 10 \cdot \log_{10} \left( \frac{\Lambda}{\ell} \right) \quad [\text{dB at length}] \quad (51)$$

With Equations (45) and (51), we have the complete set of length correction formulae for near-end and far-end cross-talk, absolutely required for any channel modelling.

## 5 Using traditional cross-talk length correction formulae

### 5.1 Background (see [4])

Using Equation (42) and Equation (43), we get

$$NEXT(\ell, f) = a + b \cdot \log_{10}(f) - 10 \cdot \log_{10} \left( 1 - e^{-4 \cdot \alpha \cdot \ell} \right) \quad [\text{dB at Length } \ell] \quad (52)$$

$$NEXT(\ell, f) = a + b \cdot \log_{10}(f) - 10 \cdot \log_{10} \left( 1 - 10^{-\frac{\alpha \cdot \ell}{5}} \right) \quad [\text{dB at Length } \ell] \quad (53)$$

where

$$a = -10 \cdot \log_{10} \left( \frac{C_N}{k_1} \right)$$

$$b = -15$$

<sup>6</sup> We could measure also the FEXT directly, with a real network analyzer, not one with an S-parameter test-set incorporated into it. The author was made aware of this nevertheless obvious fact by Eric Lawrence, today Nexans, and used this method also, though the calibration is a bit awkward.

then

$$\Delta_1 = -10 \cdot \log_{10} \left( 1 - e^{-4 \cdot \alpha \cdot \ell} \right) \quad (54)$$

$$\Delta_2 = -10 \cdot \log_{10} \left( 1 - 10^{-\frac{\alpha \cdot \ell}{5}} \right) \quad (55)$$

With normally used attenuation formula for data grade cables, we have

$$\alpha = k_1 \cdot \sqrt{f} + k_2 \cdot f + \frac{k_3}{\sqrt{f}} \quad [\text{dB}] \quad (56)$$

We get then

$$\Delta_1(f, \ell) = -10 \cdot \log_{10} \left( 1 - e^{-\frac{\ell}{5 \cdot \log(e)} \cdot \left( k_1 \cdot \sqrt{f} + k_2 \cdot f + \frac{k_3}{\sqrt{f}} \right)} \right) \quad [\text{Neper at } f, \ell] \quad (57)$$

$$\Delta_2(f, \ell) = -10 \cdot \log_{10} \left( 1 - 10^{-\frac{\ell}{5} \cdot \left( k_1 \cdot \sqrt{f} + k_2 \cdot f + \frac{k_3}{\sqrt{f}} \right)} \right) \quad [\text{dB at } f, \ell] \quad (58)$$

Equation (58) indicates the frequency and length dependency of NEXT.

## 5.2 Example (see [5], [6]) Length and frequency dependency of direct near-end crosstalk attenuation

Assuming uncorrelated direct crosstalk coupling with length the near-end crosstalk, power ratio is [6]

$$\frac{p_n}{p_o} = K_n f^{3/2} \left[ 1 - e^{-2(\alpha_1 + \alpha_2)L} \right] \quad (59)$$

and the crosstalk attenuation in dB

$$NEXT = 10 \lg \frac{p_o}{p_n} = -10 \lg K_o - 15 \lg \frac{f}{f_o} - 10 \lg \left[ 1 - e^{-2(\alpha_1 + \alpha_2)L} \right] \quad (60)$$

where

- $p_o$  is the sending power of the disturbing line;
- $p_n$  is the received near-end crosstalk power of the disturbed line;
- $k_n$  is the uncorrelated near-end crosstalk coupling of long lines;
- $K_o$  is the uncorrelated near-end crosstalk coupling of long lines at a frequency  $f_o$ ;
- $\alpha_1$  and  $\alpha_2$  are the attenuation coefficients of the disturbing and disturbed line in Np;

$L$  is the length of the lines;  
 $f$  is the frequency.

$$\Delta A_1 = 15 \lg \frac{f}{f_0} \quad (61)$$

is the frequency depending part,

$$\Delta A_2 = 10 \lg \left[ 1 - e^{-2(\alpha_1 + \alpha_2)L} \right] \quad (62)$$

is the attenuation and length depending part and

$-10 \lg K_0$  is the crosstalk attenuation of the long lines at the frequency  $f_0$ .

The theory was developed for telephone lines, which are long by nature and the third term  $A_2$  can normally be regarded as negligible.

In Tables 1 and 2 and Figures 3 to 5, the length and frequency dependencies of a category 6<sub>A</sub> cable have been studied.

The maximum attenuation in dB/100 m of category 6<sub>A</sub> is

$$\alpha[100\text{m}] = 1,820\sqrt{f} + 0,0091f + 0,250/\sqrt{f} \quad (63)$$

In Figure 3, the length and frequency dependent term  $\Delta A_2$  has been shown as a function of length with the frequency as a parameter and in Figure 4 as a function of frequency with the length as a parameter.

It can be clearly seen that for short lengths and low frequencies, the term  $\Delta A_2$  is significant. E.g. for 5 m and 100 MHz, it is about 4,5 dB and first above 20 m  $\Delta A_2$  becomes below 1 dB. For 1 m and 100 MHz,  $\Delta A_2$  is about 11 dB. For the whole studied frequency range 20 MHz to 500 MHz, the cable becomes "long" at about 100 m and for shorter cables, the third term in Equation (60), i.e.  $\Delta A_2$  of Equation (62), has to be taken into consideration.

Table 1 – Delta  $A_2$  as a function of length or frequency, the other being a parameter

$L[m] / f [MHz]$	20	50	100	200	500				
$\alpha$ [Np]	0,009634	0,015364	0,022005	0,031713	0,052046				
1	14,22485	12,24722	10,74387	9,239594	7,259733				
2	11,29742	9,368316	7,920501	6,496019	4,678015		$\alpha$ 20 MHz	0,009634	Np
5	7,563412	5,774918	4,485119	3,282069	1,891815				
10	4,951379	3,380798	2,326184	1,434219	0,578427		a=	1,82	
20	2,697738	1,503092	0,819553	0,357882	0,068063		b=	0,0091	
50	0,68349	0,205857	0,053593	0,007649	0,000131		c=	0,25	
100	0,093084	0,009318	0,000653	1,34E-05	3,95E-09				
			Delta $A_2$						

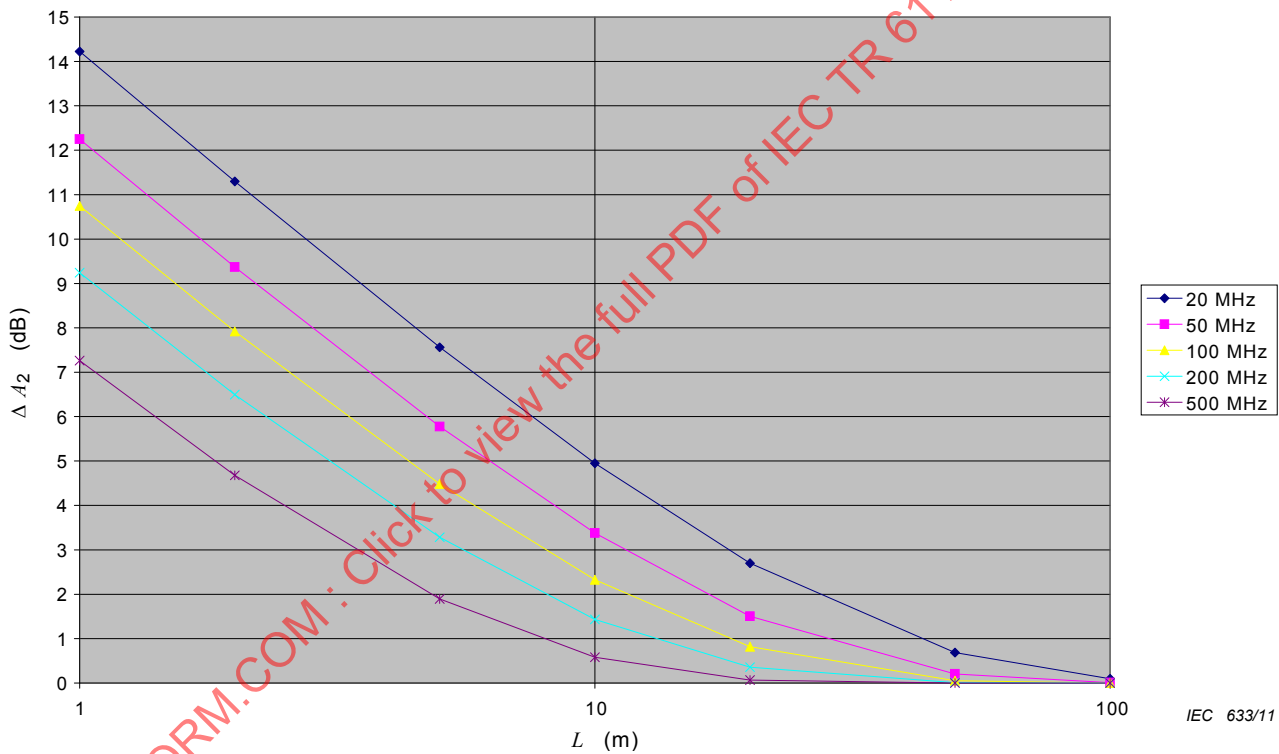


Figure 3 – Delta  $A_2$  at different frequencies as a function of length

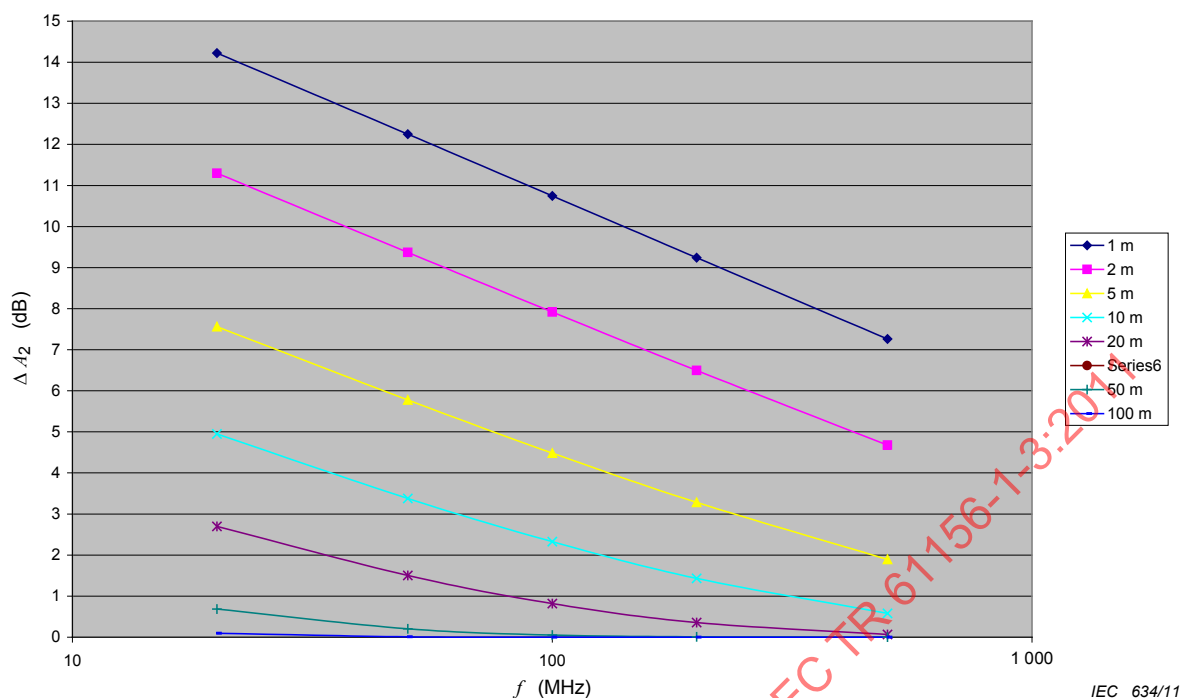
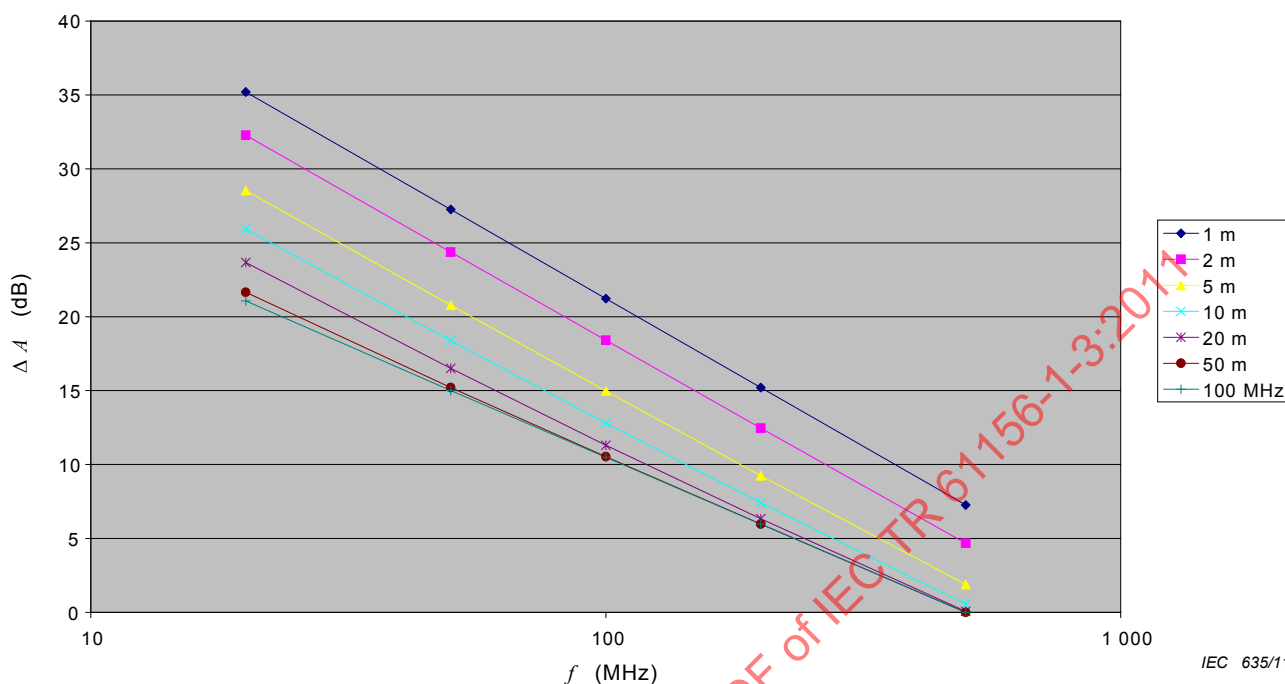


Figure 4 – Delta  $A_2$  for different lengths as a function of frequency

Table 2 – Delta  $A$  as a function of frequency (= Delta  $A_1$  + Delta  $A_2$ )

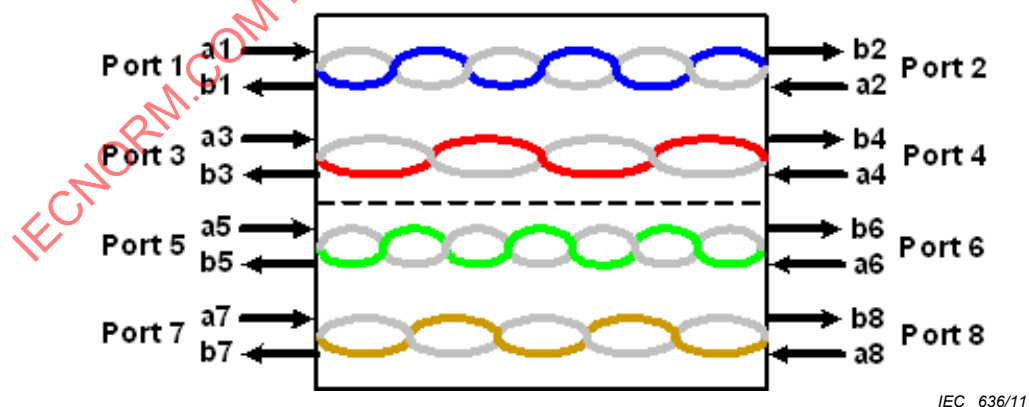
$L[\text{m}] / f [\text{MHz}]$	20	50	100	200	500				
$\alpha$ [Np]	0,009634	0,0153637	0,0220053	0,031713	0,052046				
1	35,19395	27,247219	21,22842	15,20869	7,259733				
2	32,26652	24,368316	18,405051	12,46512	4,678015		$\alpha$ 20 MHz	0,009634	Np
5	28,53251	20,774918	14,969669	9,251169	1,891815				
10	25,92048	18,380798	12,810734	7,403319	0,578427	Cat 6A	a=	1,82	
20	23,66684	16,503092	11,304103	6,326982	0,068063		b=	0,0091	
50	21,65259	15,205857	10,538143	5,97675	0,000131		c=	0,25	
100	21,06218	15,009318	10,485203	5,969114	3,95E-09				
							$f_0$ =	500	MHz
Delta $A$ =	Delta $A_1$ +	Delta $A_2$					Delta $A_1$ =	–	$15 \lg f/f_0$



**Figure 5 – Delta  $A$  for different lengths as a function of frequency  
(= Delta  $A_1$  + Delta  $A_2$ )  $f_0 = 500$  MHz**

Figure 5 shows that the frequency dependency of Delta  $A = \text{Delta } A_1 + \text{Delta } A_2$ . For “long” cables the frequency dependency is 15 dB/dec and for cables shorter than 5 m about 20 dB/dec.

## 6 On length concatenation of measured cables, using scattering and scattering transfer parameters, see informative reference [7]



**Figure 6 – Typical port assignment resulting out of the numbering of the VNA measuring ports**

If the port assignment according to Figure 6 is used, the incident and reflected waves may be arranged such as to create sub-matrices, which correspond to those of a four port. This is shown schematically in the following matrices, where  $S_1 = S_2$ .

$$S_1 = \begin{pmatrix} \text{RL}_{11} & \gamma_{12} & \text{NE}_{13} & \text{FE}_{14} & \text{NE}_{15} & \text{FE}_{16} & \text{NE}_{17} & \text{FE}_{18} \\ \gamma_{21} & \text{RL}_{22} & \text{FE}_{23} & \text{NE}_{24} & \text{FE}_{25} & \text{NE}_{26} & \text{FE}_{27} & \text{NE}_{28} \\ \text{NE}_{31} & \text{FE}_{32} & \text{RL}_{33} & \gamma_{34} & \text{NE}_{35} & \text{FE}_{36} & \text{NE}_{37} & \text{FE}_{38} \\ \text{FE}_{41} & \text{NE}_{42} & \gamma_{43} & \text{RL}_{44} & \text{FE}_{45} & \text{NE}_{46} & \text{FE}_{47} & \text{NE}_{48} \\ \text{NE}_{51} & \text{FE}_{52} & \text{NE}_{53} & \text{FE}_{54} & \text{RL}_{55} & \gamma_{56} & \text{NE}_{57} & \text{FE}_{58} \\ \text{FE}_{16} & \text{NE}_{62} & \text{FE}_{63} & \text{NE}_{64} & \gamma_{65} & \text{RL}_{66} & \text{FE}_{67} & \text{NE}_{68} \\ \text{NE}_{71} & \text{FE}_{72} & \text{NE}_{73} & \text{FE}_{74} & \text{NE}_{75} & \text{FE}_{76} & \text{RL}_{77} & \gamma_{78} \\ \text{FE}_{81} & \text{NE}_{82} & \text{FE}_{83} & \text{NE}_{84} & \text{FE}_{85} & \text{NE}_{86} & \gamma_{87} & \text{RL}_{88} \end{pmatrix} \quad (64)$$

The extreme symmetry should be noted after the swapping of rows and columns, yielding mathematically exactly the same matrix:

$$S_2 = \begin{pmatrix} \text{RL}_{11} & \text{NE}_{13} & \text{NE}_{15} & \text{NE}_{17} & \gamma_{12} & \text{FE}_{14} & \text{FE}_{16} & \text{FE}_{18} \\ \text{NE}_{31} & \text{RL}_{33} & \text{NE}_{35} & \text{NE}_{37} & \text{FE}_{32} & \gamma_{34} & \text{FE}_{36} & \text{FE}_{38} \\ \text{NE}_{51} & \text{NE}_{53} & \text{RL}_{55} & \text{NE}_{57} & \text{FE}_{52} & \text{FE}_{54} & \gamma_{56} & \text{FE}_{58} \\ \text{NE}_{71} & \text{NE}_{73} & \text{NE}_{75} & \text{RL}_{77} & \text{FE}_{72} & \text{FE}_{74} & \text{FE}_{76} & \gamma_{78} \\ \gamma_{21} & \text{FE}_{52} & \text{FE}_{53} & \text{FE}_{54} & \text{RL}_{22} & \text{NE}_{24} & \text{NE}_{26} & \text{NE}_{28} \\ \text{FE}_{16} & \gamma_{43} & \text{FE}_{63} & \text{FE}_{64} & \text{NE}_{42} & \text{RL}_{44} & \text{NE}_{46} & \text{NE}_{48} \\ \text{FE}_{71} & \text{FE}_{72} & \gamma_{65} & \text{FE}_{74} & \text{NE}_{62} & \text{NE}_{64} & \text{RL}_{66} & \text{NE}_{68} \\ \text{FE}_{81} & \text{FE}_{82} & \text{FE}_{83} & \gamma_{87} & \text{NE}_{82} & \text{NE}_{84} & \text{NE}_{86} & \text{RL}_{88} \end{pmatrix} \quad (65)$$

It should be noted furthermore that the matrices shown in Equation (64) and Equation (65) are symbolic matrices only, as they are indicated in their power ratio parameters, and not in their corresponding – and for any mathematical treatment required – voltage ratios.

In Figure 7, we have the schematics of a comparable  $2n \times 2n$  multiport, indicating the incident and reflected waves for the corresponding sub-matrices.