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Damping materials — Graphical presentation of the complex modulus

Matériaux amortissants — Représentation graphique du module complexe

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International Standard ISO 10112 was prepared by Technical Committee ISO/TC 108, *Mechanical vibration and shock*.

Annex A of this International Standard is for information only.

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Introduction

Damping is one potential approach to reducing vibration levels in a structural system. Damping is the dissipation of vibratory energy by converting it into heat, as distinguished from transporting it to another part of the system. When the damping is due to internal energy dissipation within a material which is part of the structural system, and when the damping is of engineering significance, the material is called a vibration damping material. The energy dissipation is due to molecular or crystal-lattice interactions and can be measured in terms of the stress/strain hysteresis loop of the vibration damping material. Other possible sources of damping, such as plastic deformations in the joints, relative slip at joints, air pumping in the joints, acoustic radiation of energy, eddy current losses, etc., are not covered in this International Standard.

The mechanical properties of most damping materials depend on frequency, temperature and strain amplitude at large strains; since this International Standard is restricted to linear behaviour, it does not cover the strain amplitude effect.

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Damping materials — Graphical presentation of the complex modulus

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1 Scope

This International Standard establishes the graphical presentation of the complex modulus of viscoelastic vibration damping materials which are macroscopically homogeneous, linear and thermorheologically simple. The complex modulus may be the shear modulus, Young's modulus, bulk modulus, longitudinal wave propagation modulus, or Lamé modulus. This graphical presentation is convenient and sufficiently accurate for many vibration damping materials.

The preferred nomenclature (parameters, symbols and definitions) is also given.

The primary purpose of this International Standard is to improve communication among the diverse technological fields concerned with vibration damping materials.

2 Nomenclature

2.1 Complex modulus

The operator form of the constitutive equation for the linear, isothermal, isotropic, macroscopically homogeneous, thermorheologically simple [see equation (7)] viscoelastic material being deformed in shear is defined [1] as

$$P(p_R)\tau(t) = Q(p_R)\gamma(t) \quad \ldots (1)$$

where

$\tau(t)$ is the shear stress;

$\gamma(t)$ is the shear strain;

$P(p_R)$ and $Q(p_R)$ are polynomials in p_R.

The operator p_R is defined as

$$p_R = d/dt_R \quad \ldots (2)$$

The reduced time differential dt_R is defined as

$$dt_R = dt/\alpha_T(T) \quad \ldots (3)$$

where

t is time, in seconds;

$\alpha_T(T)$ is the dimensionless temperature shift function [2] dependent on temperature, T, in kelvin.

The Fourier transform (f.t.) of equation (1) leads to the definition of G, the complex shear modulus valid for steady-state sinusoidal stress and strain, as

$$G(j\omega_R) = \tau^*(j\omega_R)/\gamma^*(j\omega_R) = Q(j\omega_R)/P(j\omega_R) \quad \ldots (4)$$

where $\tau^*(j\omega_R)$ denotes the f.t. of $\tau(t)$.

The reduced circular frequency, ω_R, is given as

$$\omega_R = \omega\alpha_T(T) = 2\pi f_R = 2\pi f\alpha_T(T) \quad \ldots (5)$$

which is a product of ω, the circular frequency, in radians per second, and the dimensionless temperature shift function, while f_R and f denote the reduced cyclic frequency and the cyclic frequency, in hertz.

The complex shear modulus is dependent on both frequency and temperature

$$G = G(\omega, T) \quad \ldots (6)$$

If (and only if) the dependency is expressed as

$$G = G(j\omega_R) = G[j\omega\alpha_T(T)] \quad \ldots (7)$$

then the material is called thermorheologically simple (TRS). Furthermore, equations (1) to (7) apply only to linear conditions.

Alternatively, consider a viscoelastic material element which undergoes a sinusoidal shear strain [3]

$$\gamma = \gamma_A \sin \omega t \quad \ldots (8)$$

which lags the sinusoidal shear stress by the phase angle δ_G:

$$\tau = \tau_A \sin(\omega t + \delta_G) \quad \ldots (9)$$

The sinusoidal strain and stress may be represented in complex notation as

$$\gamma^* = \gamma_A e^{j\omega t} \quad \ldots (10)$$

$$\tau^* = \tau_A e^{j(\omega t + \delta_G)} \quad \ldots (11)$$

The complex shear modulus, G, may be equivalently defined as

$$\begin{aligned} G &= \tau^*/\gamma^* = \tau_A e^{j\delta_G}/\gamma_A = G_M e^{j\delta_G} \\ &= G_M \cos \delta_G(1 + j \tan \delta_G) = G_R + jG_I \\ &= G' + jG'' = G_R(1 + j\eta_G) \end{aligned} \quad \ldots (12)$$

where

G_M is the magnitude of the shear modulus;

$G_R = G'$ is the real (storage) modulus;

$G_I = G'' = G_R\eta_G$ is the imaginary (loss) modulus;

$\eta_G = \tan \delta_G$ is the material loss factor in shear.

The concept holds for one-, two- and three-dimensional states of stress and strain [2]. Developments similar to the above apply to Young's modulus, E, to the bulk modulus, K, to the Lamé modulus, λ, and to the longitudinal wave propagation modulus $W = \lambda + 2G$.

A thermorheologically simple material is a material for which the complex modulus may be expressed as a complex valued function of one independent variable, namely reduced frequency, to represent its variation with both frequency and temperature.

NOTE 1 Sometimes, the real modulus and the material loss factor are treated as independent functions of reduced frequency; while this can facilitate satisfactory applications, it is a conceptual error.

The complex modulus evaluated at a given temperature and a given frequency represents both the magnitude and the phase relationships between sinusoidal stress and strain.

2.2 Data check

It is presumed in this International Standard that a set of valid complex modulus data (e.g., tables 1 and 2) has been obtained in accordance with good practice (see, for example, ref. [4]). It is recommended that each set of data be routinely and carefully scrutinized. As a minimum, the lg η_G *versus* lg G_M should be plotted (e.g., figure 1). If the set of data represents a thermorheologically simple material, if an adjustment of modulus for temperature and density is not appropriate and if the set of data has no scatter, the set of data will plot as a curve of vanishing width.

Each point along the arc of the curve corresponds to a unique value of reduced frequency [see equation (6)]. However, this is not considered in this plot. The material loss factor and the modulus magnitude are cross-plotted, and the reduced frequency, temperature, and frequency parameters do not occur explicitly. No part of any scatter in this plot can be attributed to an imperfect temperature shift function.

The loss factor *versus* modulus magnitude logarithmic plot can reveal valuable information regarding scatter of the experimental data. The width of the band of data, as well as the departure of individual points from the centre of the band, are indicative of scatter. Acceptable scatter depends on the application. Nothing is revealed about the accuracy of the temperature and frequency measurements or about any systematic error.

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Table 1 — Complex modulus data

				α_T model			
NALF	NA	$A(1)$	$A(2)$	$A(3)$	$A(4)$	$A(5)$	$A(6)$
0	0						
				Complex modulus model			
NVEM	NB	$B(1)$	$B(2)$	$B(3)$	$B(4)$	$B(5)$	$B(6)$
11	9	5,70	212	176	0,662	4,510 × 10^{-2}	3,000 × 10^{-2}
		$B(7)$	$B(8)$	$B(9)$	$B(10)$	$B(11)$	$B(12)$
		0,410	0,257	3,65			

Table 2 — Complex modulus data as a function of temperature and frequency

Temperature (K)	Frequency (Hz)	G_R (MPa)	η_G	G_I (MPa)	$\alpha_T(T)$
254,2	7,800	244,0	0,130 0	31,72	2,795 6 × 10^{4} T
254,2	15,60	252,0	0,114 0	28,73	2,795 6 × 10^{4} T
254,2	31,20	260,0	9,970 0 × 10^{-2}	25,92	2,795 6 × 10^{4} T
254,2	62,50	266,0	9,410 0 × 10^{-2}	25,03	2,795 6 × 10^{4} T
254,2	125,0	275,0	9,470 0 × 10^{-2}	26,04	2,795 6 × 10^{4} T
254,2	250,0	281,0	7,210 0 × 10^{-2}	20,26	2,795 6 × 10^{4} T
254,2	500,0	292,0	8,160 0 × 10^{-2}	23,83	2,795 6 × 10^{4} T
254,2	1000	337,0	7,030 0 × 10^{-2}	23,69	2,795 6 × 10^{4} T
273,2	7,800	77,30	0,623 0	48,16	33,94 T
273,2	15,60	96,50	0,541 0	52,21	33,94 T
273,2	31,20	119,0	0,456 0	54,26	33,94 T
273,2	62,50	140,0	0,385 0	53,90	33,94 T
273,2	125,0	162,0	0,333 0	53,95	33,94 T
273,2	250,0	185,0	0,259 0	47,92	33,94 T
273,2	500,0	206,0	0,232 0	47,79	33,94 T
273,2	1000	242,0	0,202 0	48,88	33,94 T
283,2	7,800	22,90	0,892 0	20,43	2,825 T
283,2	15,60	30,70	0,869 0	26,68	2,825 T
283,2	31,20	42,10	0,813 0	34,23	2,825 T
283,2	62,50	57,90	0,731 0	42,32	2,825 T
283,2	125,0	77,30	0,645 0	49,86	2,825 T
283,2	250,0	101,0	0,532 0	53,73	2,825 T
283,2	500,0	126,0	0,461 0	58,09	2,825 T
283,2	1000	161,0	0,388 0	62,47	2,825 T
292,2	7,800	11,10	0,745 0	8,270	0,644 6 T
292,2	15,60	14,00	0,818 0	11,45	0,644 6 T
292,2	31,20	18,30	0,869 0	15,90	0,644 6 T
292,2	62,50	24,80	0,889 0	22,05	0,644 6 T
292,2	125,0	34,60	0,875 0	30,27	0,644 6 T
292,2	250,0	48,90	0,786 0	38,44	0,644 6 T
292,2	500,0	68,00	0,703 0	47,80	0,644 6 T
292,2	1000	94,80	0,611 0	57,92	0,644 6 T
303,2	7,800	7,800	0,552 0	4,306	0,194 1 T
303,2	15,60	9,560	0,661 0	6,319	0,194 1 T
303,2	31,20	12,10	0,762 0	9,220	0,194 1 T
303,2	62,50	16,10	0,856 0	13,78	0,194 1 T
303,2	125,0	22,30	0,912 0	20,34	0,194 1 T
303,2	250,0	30,70	0,874 0	26,83	0,194 1 T
303,2	500,0	45,60	0,824 0	37,57	0,194 1 T
303,2	1000	65,40	0,747 0	48,85	0,194 1 T
313,2	7,800	6,000	0,351 0	2,106	4,308 8 × 10^{-2} T
313,2	15,60	6,800	0,448 0	3,046	4,308 8 × 10^{-2} T
313,2	31,20	7,940	0,553 0	4,391	4,308 8 × 10^{-2} T
313,2	62,50	9,520	0,661 0	6,293	4,308 8 × 10^{-2} T
313,2	125,0	11,80	0,775 0	9,145	4,308 8 × 10^{-2} T
313,2	250,0	15,30	0,845 0	12,93	4,308 8 × 10^{-2} T
313,2	500,0	20,80	0,897 0	18,66	4,308 8 × 10^{-2} T
313,2	1000	30,20	0,894 0	27,00	4,308 8 × 10^{-2} T

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Temperature (K)	Frequency (Hz)	G_R (MPa)	η_G	G_I (MPa)	$\alpha_T(T)$
333,2	7,800	5,000	0,110 0	0,550 0	$5{,}083\,1 \times 10^{-3}\ T$
333,2	15,60	5,200	0,152 0	0,790 4	$5{,}083\,1 \times 10^{-3}\ T$
333,2	31,20	5,480	0,214 0	1,173	$5{,}083\,1 \times 10^{-3}\ T$
333,2	62,50	5,890	0,289 0	1,702	$5{,}083\,1 \times 10^{-3}\ T$
333,2	125,0	6,450	0,394 0	2,541	$5{,}083\,1 \times 10^{-3}\ T$
333,2	250,0	7,280	0,506 0	3,684	$5{,}083\,1 \times 10^{-3}\ T$
333,2	500,0	8,690	0,601 0	5,223	$5{,}083\,1 \times 10^{-3}\ T$
333,2	1000	11,00	0,657 0	7,227	$5{,}083\,1 \times 10^{-3}\ T$
353,2	7,800	4,950	$3{,}220\,0 \times 10^{-2}$	0,159 4	$7{,}650\,0 \times 10^{-4}\ T$
353,2	15,60	5,010	$4{,}510\,0 \times 10^{-2}$	0,226 0	$7{,}650\,0 \times 10^{-4}\ T$
353,2	31,20	5,090	$7{,}000\,0 \times 10^{-2}$	0,356 3	$7{,}650\,0 \times 10^{-4}\ T$
353,2	62,50	5,210	0,101 0	0,526 2	$7{,}650\,0 \times 10^{-4}\ T$
353,2	125,0	5,340	0,158 0	0,843 7	$7{,}650\,0 \times 10^{-4}\ T$
353,2	250,0	5,620	0,236 0	1,326	$7{,}650\,0 \times 10^{-4}\ T$
353,2	500,0	6,070	0,317 0	1,924	$7{,}650\,0 \times 10^{-4}\ T$
353,2	1000	7,240	0,288 0	2,085	$7{,}650\,0 \times 10^{-4}\ T$

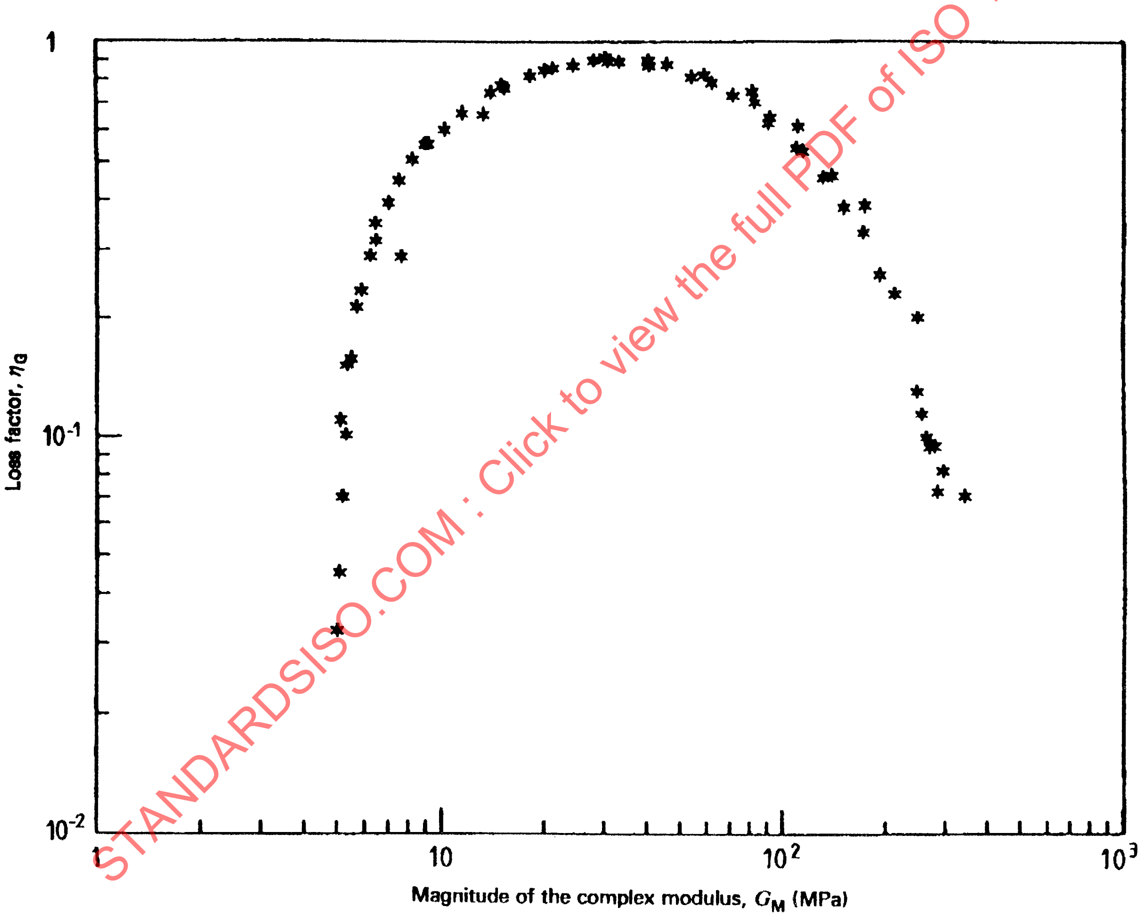

Figure 1 — Data quality check

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2.3 Temperature shift function

The set of complex modulus data itself implicitly defines the temperature shift function $\alpha_T(T)$, provided the experimental ranges of temperature and frequency are adequate. It is assumed that a single temperature shift function is applicable.

It is recommended that the following three functions be plotted for the experimental range of temperature (e.g., figure 2) because

a) the temperature shift function, $\alpha_T(T)$, has historically had a central role;

b) its slope, $\mathrm{d}(\lg \alpha_T)/\mathrm{d}T$, is the crucial feature that causes data to be correctly shifted; and

c) the apparent activation energy [2], ΔH_A, is of interest and is given by

$$\Delta H_A = 2{,}303RT^2 \mathrm{d}(\lg \alpha_T)/\mathrm{d}T \qquad \ldots (13)$$

where R is the gas constant

$$R = 0{,}00828 \text{ N·km/g·mol·K} \qquad \ldots (14)$$

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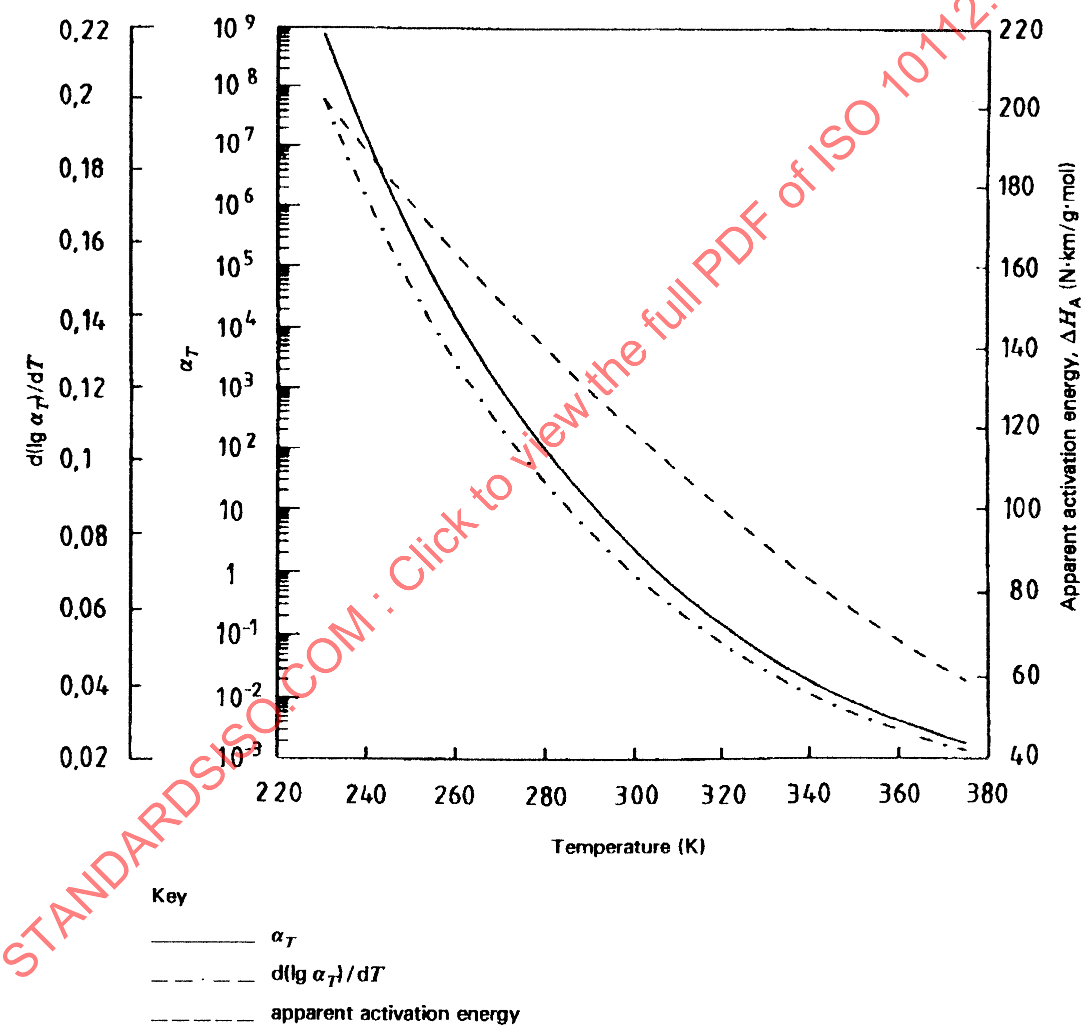

Figure 2 — Temperature shift function and its properties

3 Graphical presentation

3.1 Reduced frequency plot

A set of complex modulus data is presented in figure 3. A logarithmic scale is shown along the vertical axis for the real and the imaginary modulus components, in megapascals (MPa), and for the dimensionless loss factor. The logarithmic scale along the horizontal axis is reduced cyclic frequency, f_R, in hertz.

The reduced frequency for the i^{th} experimental point, f_{Ri}, is given by

$$f_{Ri} = f_i \alpha_T(T_i) \qquad \ldots (15)$$

where

f_i is the experimental frequency;

T_i is the experimental temperature.

3.1.1 Jones temperature lines

In figure 3 the vertical logarithmic scale on the right is the cyclic frequency, in hertz. The non-uniformly spaced diagonal constant temperature lines, together with the horizontal reduced frequency axis and the vertical frequency axis, provide a temperature-frequency-reduced frequency nomogram [5].

The logarithmic form of equation (5) is

$$\lg f_R = \lg f + \lg \alpha_T(T) \qquad \ldots (16)$$

which is the equation for the straight line in figure 3.

Values of temperature, in kelvin, at convenient intervals are chosen. The spacing of the set of constant temperature lines depends on the temperature shift function used. The range of the diagonal lines should be chosen to be the same as the experimental temperature range of data to preclude unintentional (and possibly highly erroneous) extrapolation.

Furthermore, the diagonal isotherm lines are shown as solid lines in the range of experimental frequency and as dashed lines outside the experimental range. It follows that the reduced frequency scale covers the range from the lowest temperature line and the highest frequency on the right-hand scale to the highest temperature line and the lowest frequency.

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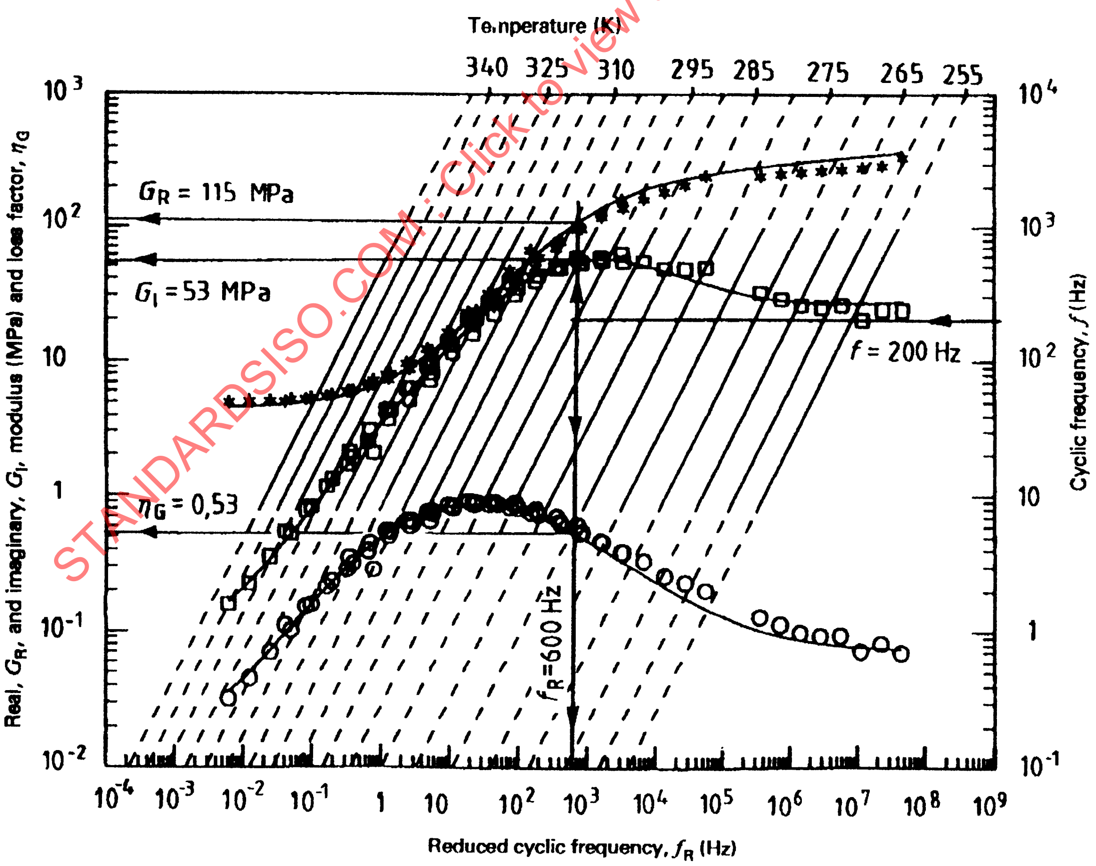

Figure 3 — Complex modulus reduced frequency plot

To follow the illustration included in figure 3, enter at the right at a frequency of 200 Hz and proceed horizontally to the intersection with the diagonal line representing 295 K. The intersection defines a reduced frequency of 600 Hz. Where a vertical line at this reduced frequency crosses the data curves, a real component of 115 MPa, an imaginary component of 53 MPa, and a loss factor of 0,53 are read on the left-hand scale.

3.1.2 Inverted-"U" plot

The same set of complex modulus data is also presented in figure 4. The left (vertical, logarithmic scale) axis is the dimensionless loss factor. The horizontal (logarithmic scale) axis is the real modulus, in megapascals (MPa).

A nomogram [6] based on equation (15) is included. To illustrate, enter the right scale at 200 Hz and proceed horizontally to the 295 K curve; from this intersection point proceed downwards to read 120 MPa on the horizontal axis, and proceed upwards to the data curve, for which intersection read a loss factor of 0,53 on the left-hand scale.

3.2 Analytical representation

While manual processing and interpretation are adequate for some purposes, computerization offers considerable efficiency. Furthermore, analytical representation of the temperature shift function and of the complex modulus provides increased efficiency for design studies. If available, analytical representations should be given (e.g., tables 3 and 4).

When graphical constructions are used in determining values for parameters used in equations or in interpretation, they shall be included (e.g. linear plots of imaginary modulus *versus* real modulus to determine intercept angles of the data curve with the real axis).

When analytical representation is used in design, care shall be taken to avoid inappropriate extrapolation.

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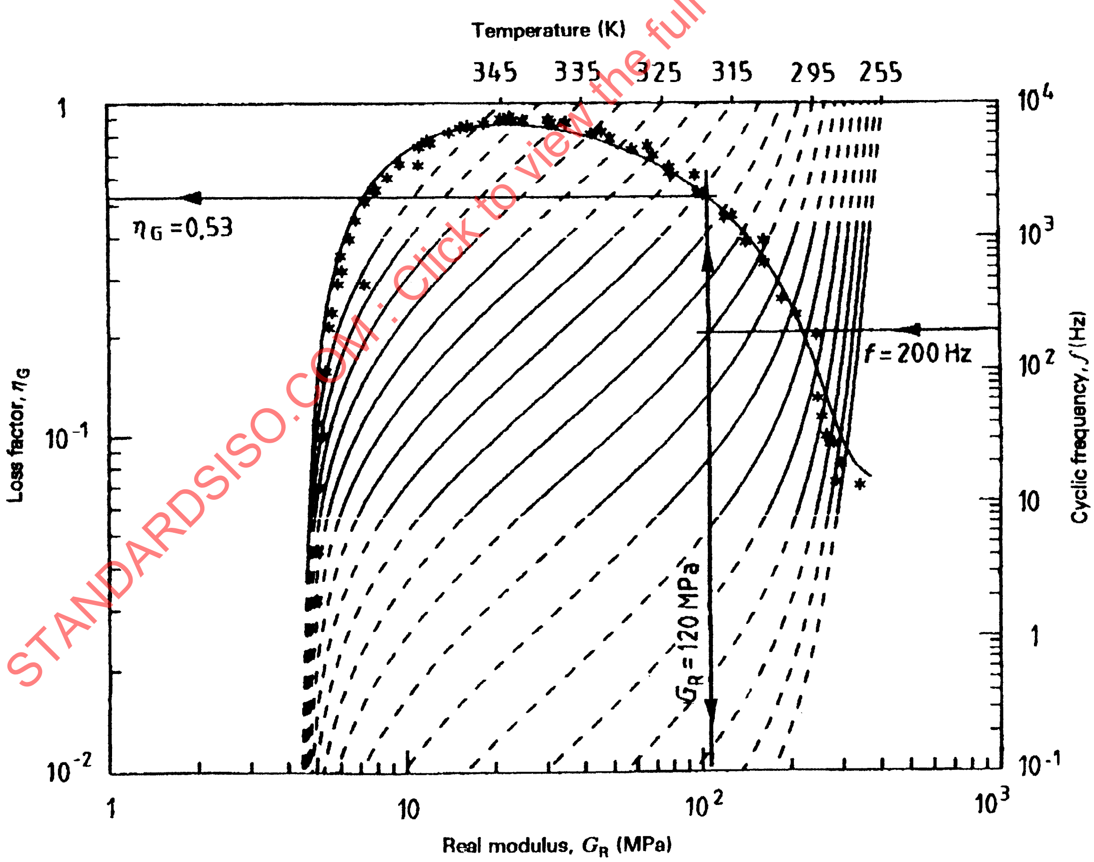

Figure 4 — Complex modulus inverted-"U" plot

Table 3 — Example of analytical representation of the temperature shift function

$$\lg \alpha_T = a(1/T - 1/T_Z) + 2{,}303(2a/T_Z - b)\lg(T/T_Z) - \left(b/T_Z - a/T_Z^2 - S_{AZ}\right)(T - T_Z)$$

$$-\mathrm{d}(\lg \alpha_T)/\,\mathrm{d}T = a(1/T - 1/T_Z)^2 + b(1/T - 1/T_Z) + S_{AZ}$$

Slope through the three points:

$$T_Z = A(1) = 290;\ S_{AZ} = A(4) = 0{,}069$$

$$T_L = A(2) = 230;\ S_{AL} = A(5) = 0{,}2$$

$$T_H = A(3) = 360;\ S_{AH} = A(6) = 0{,}04$$

$$C_A = (1/T_L - 1/T_Z)^2$$

$$C_B = 1/T_L - 1/T_Z$$

$$C_C = S_{AL} - S_{AZ}$$

$$D_A = (1/T_H - 1/T_Z)^2$$

$$D_B = 1/T_H - 1/T_Z$$

$$D_C = S_{AH} - S_{AZ}$$

$$D_E = D_B C_A - C_B D_A$$

$$a = (D_B C_C - C_B D_C)/D_E$$

$$b = (C_A D_C - D_A C_C)/D_E$$

Table 4 — Example of analytical representation of the complex modulus

$$G = [G_e(jf_R/10^{-\alpha_R} f_{RO})^{\beta_R} + G_g(jf_R/f_{RO})^{\beta_T}]/[1 + (jf_R/10^{-\alpha_F} f_{RO})^{\alpha_B \beta_T} + (jf_R/f_{RO})^{\beta_T - \beta_G}]$$

$$G_e = B(1) = 5{,}0$$

$$G_g = B(2) = 320$$

$$f_{RO} = B(3) = 410$$

$$\beta_T = B(4) = 0{,}66$$

$$\beta_G = B(5) = 0{,}01$$

$$\beta_R = B(6) = 0{,}005$$

$$\alpha_F = B(7) = 0{,}52$$

$$\alpha_B = B(8) = 0{,}58$$

$$\alpha_R = B(9) = 3$$

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